Cluster separability, indispensability of detectors and quantum measurement problem

P. Hájíček
Institute for Theoretical Physics
University of Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland
hajicek@itp.unibe.ch

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Abstract

Careful analysis of cluster separability opens a way to a completely new understanding of preparation and registration procedures for microsystems: they are changes of separation status. An important observation is that quantum mechanics does not specify what happens to systems when they change their separation status. New rules that close this gap can therefore be added without disturbing the logic of quantum mechanics. Another important observation is that registration apparatuses for microsystems must contain detectors and that their readings are signals from detectors. This leads to further restrictions on measurability of observables, especially for macroscopic quantum systems. Beltrametti-Cassinelli-Lahti model is used to show how this approach leads to solution of the measurement problem.

Discussions about the nature of quantum measurement were started already by founding fathers of the theory, persisted throughout and seem even to amplify at the present time. An example is the quantum decoherence theory [1, 2], another the superselection sectors approach [3, 4], etc. However, the problem is far from being satisfactorily solved. For example, Refs. [5, 6, 7] analyse the well-known shortcomings. We adopt the definition of the problem and the proof that it is far from being solved from [7] and give a short account of a new approach to quantum measurement problem based on preliminary results of [8, 9, 10]. For the time being, it is applicable to measurements on microscopic quantum systems. Large systems such as strong laser signals or BECs will be studied later.

The fact that particles of the same type are indistinguishable in quantum mechanics implies that experiments with one particle are disturbed by another particle of the same type, even if it were prepared independently, far away from the first.

Suppose that state $\psi(\vec{x}_1)$ of system $\mathcal{S}_1$ is prepared in our laboratory as if no other system of this type existed. Next, let state $\phi(\vec{x}_2)$ of system $\mathcal{S}_2$ of the same type be prepared simultaneously in a remote laboratory. Then the state of the composite system must be

$$\Psi(\vec{x}_1, \vec{x}_2) = \nu\big(\psi(\vec{x}_1)\phi(\vec{x}_2) \pm \phi(\vec{x}_1)\psi(\vec{x}_2)\big) \ , \tag{1}$$

where ν is a normalisation factor. It follows that the state of $\mathcal{S}_1$ is not $\psi(\vec{x}_1)$ even if this state has been prepared in our laboratory but that it depends on what has been done somewhere in the world in another laboratory.

Similarly, if the registration on $\mathcal{S}_1$ corresponding to an observable $a(\vec{x}_1; \vec{x}_1')$ of $\mathcal{S}_1$ is now performed in our laboratory, it is equally possible that the registration is made on $\mathcal{S}_1$ or $\mathcal{S}_2$ and both can make a contribution to the outcome. Hence, the correct observable that describes such a registration is operator

$$A(\vec{x}_1, \vec{x}_2; \vec{x}_1', \vec{x}_2') = a(\vec{x}_1; \vec{x}_1')\delta(\vec{x}_2 - \vec{x}_2') + a(\vec{x}_2; \vec{x}_2')\delta(\vec{x}_1 - \vec{x}_1') \ . \tag{2}$$

There seems to be no control of states that are prepared anywhere in the world and the different possibilities have different measurable consequences. For example, the position operator of $\mathcal{S}_1$ is $a(\vec{x}_1; \vec{x}_1') = \vec{x}_1\delta(\vec{x}_1 - \vec{x}_1')$ and suppose that the position is registered. Then, the average in the prepared state is

$$\int d^3x_1 \vec{x}_1 \psi^*(\vec{x}_1)\psi(\vec{x}_1) \ .$$

On the other hand, the preparation of $\mathcal{S}_2$ leads to a different value:

$$\int d^3x_1 d^3x_2 d^3x_1' d^3x_2' \Psi^*(\vec{x}_1', \vec{x}_2') A(\vec{x}_1, \vec{x}_2; \vec{x}_1', \vec{x}_2') \Psi(\vec{x}_1, \vec{x}_2)$$
$$= \int d^3x_1 \vec{x}_1 \psi^*(\vec{x}_1)\psi(\vec{x}_1) + \int d^3x_1 \vec{x}_1 \phi^*(\vec{x}_1)\phi(\vec{x}_1) \ .$$

Cluster separability (see, e.g., [11], P. 128) was invented to deal with these problems. The key notion is that of D-local observable:

Definition 1 *Let $a(\vec{x}_1;\vec{x}'_1)$ be an observable of $\mathcal{S}_1$, let D be the domain of $\vec{x}_1$ inside our laboratory and let*

$$\int d^3x_1 a(\vec{x}_1;\vec{x}'_1)f(\vec{x}_1) = \int d^3x'_1 a(\vec{x}_1;\vec{x}'_1)f(\vec{x'}_1) = 0 \tag{3}$$

*if $(\mathrm{supp}\, f)\cap D = \emptyset$, where f is a test function. Let us call such operators D-*local.

Let us assume that $(\mathrm{supp}\,\psi)\subset D$ and $(\mathrm{supp}\,\phi)\cap D = \emptyset$. If $\mathcal{S}_2$ has been prepared and the D-local observable $a_D(\vec{x}_1,\vec{x}'_1)$ is used instead of $a(\vec{x}_1;\vec{x}'_1)$ in formula (2) defining operator A_D instead of A and we obtain

$$\int_D d^3x_1 \int_D d^3x'_1 \int_D d^3x_2 \int_D d^3x'_2 \Psi^*(\vec{x}_1,\vec{x}_2)A_D(\vec{x}_1,\vec{x}'_1;\vec{x}_2,\vec{x}'_2)\Psi(\vec{x}'_1,\vec{x}'_2)$$
$$= \int_{-\infty}^{\infty} d^3x_1 \int_{-\infty}^{\infty} d^3x'_1 \psi^*(\vec{x}_1)a(\vec{x}_1;\vec{x}'_1)\psi(\vec{x}'_1)$$

as if no $\mathcal{S}_2$ existed. It follows that in this case both rules for individual systems and rules for identical subsystems lead to the same results.

However, observables that are usually associated with $\mathcal{S}_1$ are not D-local. For example, the position operator $\vec{x}_1\delta(\vec{x}_1-\vec{x}'_1)$ violates the condition by large margin, as seen above. In fact, this 'observable' controls position of the system in the whole infinite space. This is utterly different from observables that can be registered in a human laboratory. Nevertheless, one can construct observables for $\mathcal{S}_1$ that are D-local and are, therefore, more realistic (see [10]).

This motivates introduction of the concept of *separation status* of a microsystem. For example, a microsystem that is alone in the Universe has a different separation status than the same microsystem in a domain D in which there are no microsystem of the same type but which is surrounded by matter containing a lot of such microsystems (assuming, of course, that supports of their states do not intersect D). Observables of the first system are the standard ones, whereas observables of the second are only the D-local ones. An extreme case of separation status for a microsystem is if its 'free' domain D shrinks to zero and the only available modus of description for it is that of identical subsystems.

We observe that the current version of quantum mechanics is a theory of systems with a fixed separation status. Let us call this restricted understanding *fixed-status quantum mechanics* (FSQM). It deals with single microsystems as if no other microsystems of the same type existed according to one set of rules and with composite systems containing many identical subsystems according to another set of rules. It

neglects the obvious relations that make such an approach in principle inconsistent. Still, the method works and the justification why it works is the cluster separability. However, FSQM might have limits and the limits have to do with changes of the separation status of a system during its time evolution.

Next, we have to look at the registration apparatuses. A careful study leads to the following observation:

Rule 1 *Any registration apparatus for microsystems must contain at least one detector and every reading of an apparatus value is a signal from a detector.*

Rule 1 seems to be obvious but it has far reaching consequences for existence of observables.

An important example are observables of macroscopic systems. In general, a macroscopic system $\mathcal{A}$ is a composite quantum system with very many different microsystem constituents. One can subdivide these microsystems into type classes. Consider first observables that concern properties of microscopic subsystem $\mathcal{S}$ of type τ. If we apply the basic rules of observable construction for systems of identical microsystems, then e.g. the position and momentum of any individual microsystem $\mathcal{S}$ are not observables of $\mathcal{A}$ but 'collective' one-particle operators such as

$$\mathsf{a}_{\mathrm{col}} = \sum_k a(\vec{x}_k; \vec{x}'_k) \ ,$$

could be, where $a(\vec{x}_k; \vec{x}'_k)$ is an operator acting on k-th subsystem of type τ. Suppose that there is an apparatus $\mathcal{B}$ suitable to measure $a(\vec{x}; \vec{x}')$ on an individually prepared system $\mathcal{S}$. One can imagine that applying $\mathcal{B}$ to $\mathcal{A}$ in some way similar to that described above would measure $\mathsf{a}_{\mathrm{col}}$ because any subsystem of type τ would automatically contribute to the result of the measurement. However, the registration apparatus was applied to individually prepared subsystems there. It follows from Rule 1 that the apparatus $\mathcal{B}$ cannot be applied to $\mathcal{A}$ in this way because none of the identical subsystems of type τ are prepared individually. Its readings are signals of its detector and for any detector to register $\mathcal{S}$, $\mathcal{S}$ must be isolated to be manipulable, have sufficient kinetic energy, etc. Hence, to measure collective observable $\mathsf{a}_{\mathrm{col}}$, we need a method that makes measurements directly on $\mathcal{A}$.

For example, let $\mathcal{A}$ be a crystal. By scattering X-rays off it, relative positions of its nuclei can be recognised. But rather than a position of individual nucleus it is a space dependence of the average nuclear density due to all nuclei. Such an average nuclear density could be obtained with the help of an operator similar to $\mathsf{a}_{\mathrm{col}}$. In general, scattering of a microsystem $\mathcal{S}'$ off a macrosystem $\mathcal{A}$ can be calculated from potential $V_k(\vec{x}, \xi, \vec{x}_k, \xi_k)$ that describes the interaction between $\mathcal{S}'$ and one of the microscopic subsystems of $\mathcal{A}$. The whole interaction Hamiltonian is then a sum

extending over all subsystems that interact with $\mathcal{S}'$,

$$\sum_k V_k(\vec{x}, \xi, \vec{x}_k, \xi_k) \ . \tag{4}$$

It is important to realise that there are very few interactions and not all of them can supply potentials useful for practical experiment.

Another example is the kinetic energy of $\mathcal{S}$. Again, the corresponding collective observable cannot be measured by the method kinetic energy is measured on individual systems of type $\mathcal{S}$. But the average of the collective observable could have the meaning of N/k_B times the temperature, where N is the number of $\mathcal{S}$-constituents of $\mathcal{A}$. Hence, a viable method to measure the average is to measure the temperature of $\mathcal{A}$. Again, this is a very special case that works only under specific conditions. Further examples have to do with other additive quantities, such as momentum and angular momentum. Total values of these quantities can be measured and they are of the form (4).

We notice, first, that an observable $a(\vec{x}_k; \vec{x}'_k)$ of a microsystem $\mathcal{S}$ can be promoted to a collective observable $\mathsf{a}_{\mathrm{col}}$ of $\mathcal{A}$ if $\mathcal{A}$ admits a direct measurement of $\mathsf{a}_{\mathrm{col}}$, which happens only in special and rare cases. Second, such a collective observable is still too 'sharp', because only some averages with rather large variances can be observed. It is impossible to obtain its single eigenvalues as results of registration (for an example, see Ref. [11], P.181). Thus, one of the consequences of Rule 1 is a principal and severe limit on mathematically well-defined quantities being observables. For more discussion, see [10].

The idea that the fixed-status quantum mechanics (FSQM) has obvious limits as well as the idea of detector indispensability lead to a considerable modification of quantum theory of measurement. The necessary changes are:

1. Preparation has a different and much greater significance than is usually assumed. Any preparation gives the prepared system $\mathcal{S}$ its objective quantum properties such as states, gemenge structures, averages and variances of observables (for extended discussion, see [8]) so that it is justified to speak of a physical object. This is what we call *quantum object*. Simultaneously, a preparation must separate $\mathcal{S}$ from the set of identical microsystems, at least approximately. The prepared state must be D-local in a suitable domain D. Only then, $\mathcal{S}$ can be viewed as an individual system and the standard notion of observable becomes applicable to it. This is justified by the idea of cluster-separability. Finally, a preparation may isolate the microsystem so that it can be individually manipulated by e.g. external fields or mater shields and registered by detectors.

2. Registration has a more specific form than is usually assumed. Any apparatus that is to register a microsystems directly contains a detector and the 'pointer' value that is read off the apparatus is a signal from the detector. We assume that each detector contains a bulk of sensitive matter (see, e.g., [12]) with which the registered microsystem is unified and changes its separation status. The change of separation status during a registration is a similar to, but a deeper change than, the so-called collapse of the wave function. Indeed, standard quantum mechanics provides no information about processes such as preparation and registration, in which the separation status of microsystems changes. Unjustified applications of standard rules to such processes leads to contradictions with experimental evidence (the so-called objectification problem, see [7]). However, one can add new rules to quantum mechanics governing such processes without violating its logic. Macrosystem $\mathcal{A}$ such as a blocking shield, a scattering target or a detector that contains microsystems indistinguishable from $\mathcal{S}$ must lie at the boundary of D. Corrections to FSQM description of the behaviour of the composed system $\mathcal{S}+\mathcal{A}$ due to a possible separation status change of $\mathcal{S}$ must be carefully chosen.

The usual method of FSQM is to specify initial states of both $\mathcal{S}$ and $\mathcal{A}$ before their interaction, choose some appropriate interaction Hamiltonian and calculate the corresponding unitary evolution of the composed system $\mathcal{S}+\mathcal{A}$ ignoring the problem with separation status change. We shall now show an example of corrections that must be done.

Let a discrete observable O of system $\mathcal{S}$ be measured. Let o_k be eigenvalues and $\{\phi_{kj}\}$ be the complete orthonormal set of eigenvectors, $\mathsf{O}\phi_{kj} = o_k\phi_{kj}$. Let the registration apparatus be a quantum system $\mathcal{A}$. Let A be a non-degenerate, discrete observable of $\mathcal{A}$ with the same eigenvalues o_k and with the complete orthonormal set of eigenvectors ψ_k, $\mathsf{A}\psi_k = o_k\psi_k$.

Let the measurement start with the preparation of $\mathcal{S}$ in state ϕ and the independent preparation of $\mathcal{A}$ in state ψ. Let $\mathcal{S}$ and $\mathcal{A}$ then interact for a finite time and let the resulting state be given by $\mathsf{U}(\phi\otimes\psi)$, where U is a unitary transformation. Unitary evolution of $\phi\otimes\psi$ and discreteness of O are definition properties of Beltrametti-Cassinelli-Lahti model of premeasurement [13]. Then, for any initial state ψ of $\mathcal{A}$, there is a set $\{\varphi_{kl}\}$ of unit vectors in the Hilbert space of $\mathcal{S}$ satisfying the orthogonality conditions $\langle\varphi_{kl}|\varphi_{kj}\rangle = \delta_{lj}$ such that U is a unitary extension of the map

$$\phi_{kl}\otimes\psi \mapsto \varphi_{kl}\otimes\psi_k \ . \tag{5}$$

For proof, see [13].

To obtain a model of measurement, it is necessary (but not sufficient) that

$$\langle\varphi_{ki}|\varphi_{lj}\rangle = \delta_{kl}\delta_{ij} \ . \tag{6}$$

Let the initial state of $\mathcal{S}$ be an arbitrary state ϕ. Decomposing ϕ into the eigenstates,

$$\phi = \sum_{kl} c_{kl}\phi_{kl} \ ,$$

we obtain from Eq. (5)

$$\mathsf{U}(\phi \otimes \psi) = \sum_{k} \sqrt{p^{\mathsf{O}}_{\phi}(o_k)}\Phi_k \otimes \psi_k \ , \tag{7}$$

where

$$\Phi_k = \frac{\sum_l c_{kl}\varphi_{kl}}{\sqrt{\langle \sum_l c_{kl}\varphi_{kl}| \sum_j c_{kj}\varphi_{kj}\rangle}}$$

and

$$p^{\mathsf{O}}_{\phi}(o_k) = \left\langle \sum_l c_{kl}\varphi_{kl} \,\middle|\, \sum_j c_{kj}\varphi_{kj} \right\rangle$$

is the probability that a registration of O performed on vector state ϕ gives the value o_k. The final state of apparatus $\mathcal{A}$ then follows from Eq. (6),

$$tr_{\mathcal{S}}[|\mathsf{U}(\phi \otimes \psi)\rangle\langle \mathsf{U}(\phi \otimes \psi)|] = \sum_j p^{\mathsf{O}}_{\phi}(o_j)|\psi_j\rangle\langle\psi_j| \ . \tag{8}$$

At the end of the registration, $\mathcal{A}$ must objectively be in one of the states ψ_j in each individual case (objectification requirement). That is, the right-hand side of Eq. (8) must be the *gemenge structure* of the state [7, 10] (some authors [5] use the term 'proper mixture' instead of 'gemenge'). However, state (8) of $\mathcal{A}$ has not the gemenge structure given by the right-hand side of Eq. (8) because of the entanglement with $\mathcal{S}$ due to state (7). The reason is that state (7) contains much more correlations between observables of $\mathcal{S}$ and $\mathcal{A}$ than just correlations between the states Φ_k and ψ_k. To measure any of these correlations, we would always need some observables of $\mathcal{S}$ that do not commute with O.

However, the assumption that the end state of $\mathcal{S} + \mathcal{A}$ is (7) seems to be an illusion. Microsystem $\mathcal{S}$ is assumed to be somewhere inside $\mathcal{A}$ at this stage and is indistinguishable from other microsystems of the same type within $\mathcal{A}$. There is always a lot of them, either because they are present in the detector(s) before the registration started or because the detector(s) becomes quickly polluted by them afterwards. Thus, the separation status of the system $\mathcal{S}$ has changed and with it also the separation status of the whole composite system $\mathcal{S}+\mathcal{A}$ has. The applications of FSQM to two systems of different separation status is different. In our case, system $\mathcal{S} + \mathcal{A}$ before the interaction is a composite one and each of the subsystems is an object having its states and observables. During and after the interaction, however, $\mathcal{S}$ ceases to be an object, becomes a part of $\mathcal{A}$ and looses all of its observables except

of O. This is a deeper change than just a change of state. Hence, the existence of most correlations that define state (7) is lost. The point is not that some observables are difficult to measure but rather that these observables do not exist at all. The only correlations that can remain are those between end states ψ_k of $\mathcal{A}$ and Φ_k of the microsystem. They define the state

$$\sum_k p_\phi^{\mathsf{O}}(o_k)|\Phi_k\rangle\langle\Phi_k| \otimes |\psi_k\rangle\langle\psi_k| \ . \tag{9}$$

This motivates the following assumption:

Rule 2 *Let discrete observable O of microsystem $\mathcal{S}$ be registered by apparatus $\mathcal{A}$ and the corresponding unitary evolution leads to the state (7) with $\mathcal{S}$ inside $\mathcal{A}$. Then the true state of $\mathcal{S}+\mathcal{A}$ and its gemenge structure are given by Eq. (9).*

The end state of $\mathcal{A}$ has then necessarily gemenge structure (8). The content of Rule 2 is that only the correlations between the states ψ_k of $\mathcal{A}$ and Φ_k of $\mathcal{S}$ survive and all other correlations between $\mathcal{A}$ and $\mathcal{S}$ are erased during the change of separation status of $\mathcal{S}+\mathcal{A}$. What survives and what is erased is uniquely determined by the Beltrametti-Cassinelli-Lahti model. In particular, states φ_{kl} are uniquely determined by initial state ψ of $\mathcal{A}$ and initial state ϕ of $\mathcal{S}$ determines states Φ_k uniquely. Thus, the additional evolution from state (7) to state (9) is non-unitary but still deterministic. Rule 2 is a new general rule which has to be added to quantum mechanics. To choose such a rule, we have to look at observations and experiments. Rule 2 is in an agreement with what is observed. For more discussion of Rule 2, see [10].

It ought to be clear from this example how our method works. Further possible questions are discussed in [10].

References

[1] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, H. D. Zeh, *Decoherence and the Appearance of Classical World in Quantum Theory*, Springer, Berlin, 1996.

[2] W. H. Zurek, Rev. Mod. Phys.,**75** (2003) 715.

[3] K. Hepp, Helvetica Phys. Acta, **45** (1972) 237.

[4] H. Primas, *Chemistry, Quantum Mechanics and Reductionism.* Springer, Berlin, 1983.

[5] B. d'Espagnat, *Veiled Reality*, Addison-Wesley, Reading, 1995.

[6] J. Bub, *Interpreting the Quantum World*, Cambridge University Press, Cabridge, UK, 1999.

[7] P. Bush, P. J. Lahti and P. Mittelstaed, The Quantum Theory of Measurement, Springer, Heidelberg, 1996.

[8] P. Hájíček and J. Tolar, Found. Phys. **39** (2009) 411.

[9] P. Hájíček, Foud. Phys. **39** (2009) 1072.

[10] P. Hájíček, arXiv:1001.1827.

[11] A. Peres, *Quantum Theory: Concepts and Methods*, Kluwer, Dordrecht, 1995.

[12] W. R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, Springer, Berlin, 1987.

[13] E. G. Beltrametti, G. Cassinelli and P. J. Lahti, J. Math. Phys. **31** (1990) 91.